Double-parton scatterings in p-A & A-A collisions at the LHC

Hard Probes 2013

Stellenbosch (South Africa) – 5th Nov. 2013

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CERN

(*) Most of the results based upon:

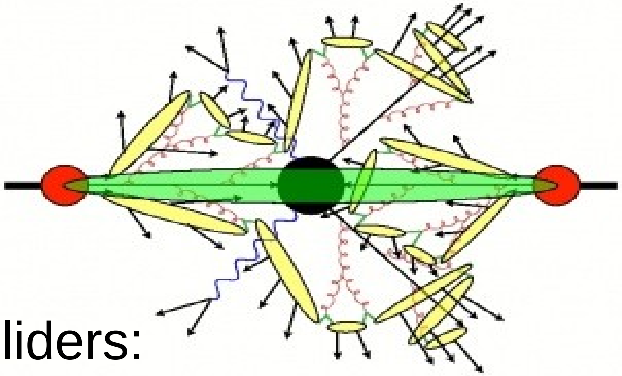
Dd'E & A.Snigirev, arXiv:1211.0197 [PLB 718 (2013) 1395]
Dd'E & A.Snigirev, arXiv:1301.5845 [PLB 727 (2013) 157]

Outline

- Introduction – Double Parton Scattering in p-p collisions:
  - Multi-parton interactions.
  - Phenomenological "pocket formula" for the DPS x-section
  - Experimental DPS searches
- Double-Parton-Scattering in p-A collisions:
  - DPS "pocket formula" x-section. Enhancement factor wrt. DPS(pp): $\times 3 \cdot A$
  - Case study: Same-sign WW in p-Pb at the LHC:
    σ(ssWW,DPS) $\approx$ 150 pb > σ(ssWWjj) $\approx$ 100 pb,
    $N_{visible}$(ssWW,DPS) $\approx$ 10 at 8.8 TeV
- Double-Parton-Scattering in A-A collisions:
  - DPS "pocket formula" x-section. Enhancement factor wrt. DPS(pp): $\times A^{3.3}/5$
  - Case study: Double-J/ψ in Pb-Pb at the LHC:
    20% (30%) of MB (central) Pb-Pb colls. Produce 2 J/ψ
    $N_{visible}(J/\psi J/\psi \rightarrow l^+l^-l^+l^-) \approx 240$ at 5.5 TeV

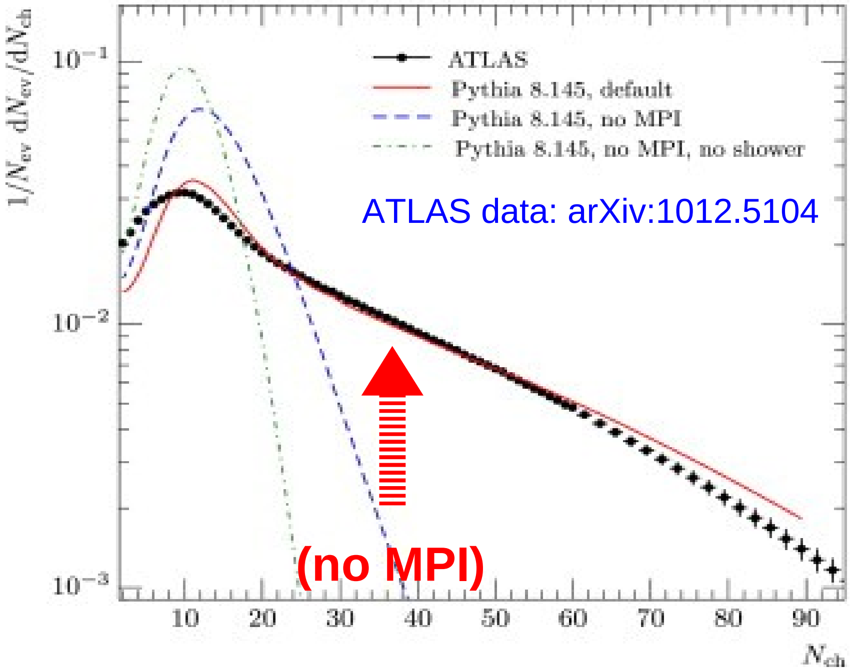
Multi-parton interactions at the LHC

- MPI are unavoidable in hadron collisions (p,Pb) = non-pointlike objects with finite transverse size and increasingly larger gluon density.

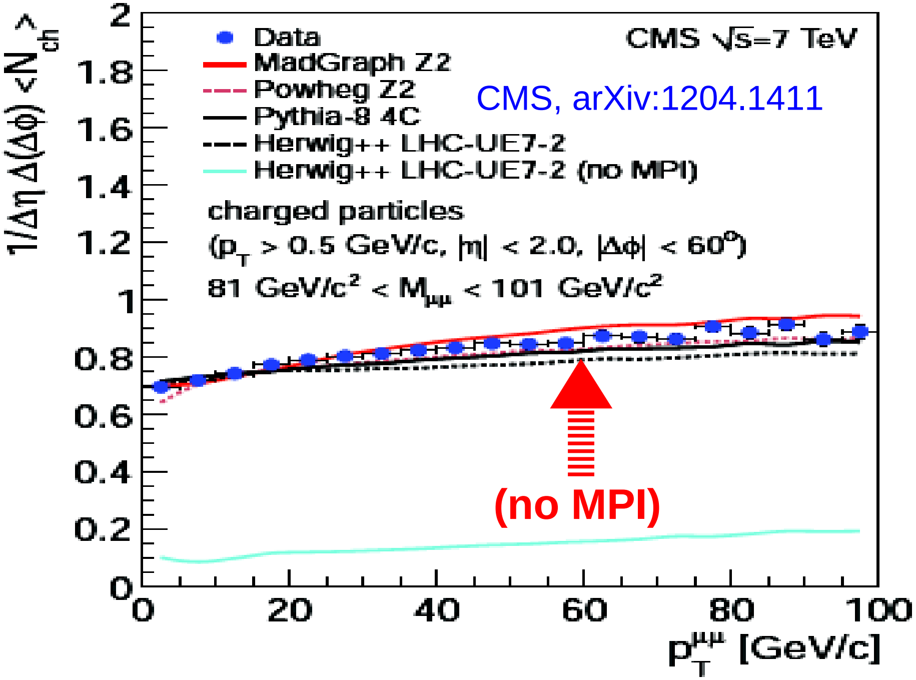


- MPI O(1-3 GeV) clearly observed in hadron colliders:

~50% of total hadron production



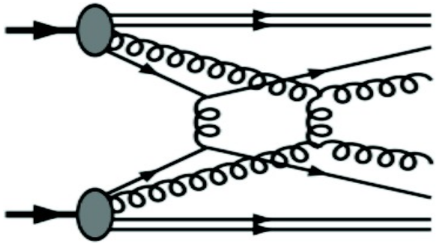
Underlying event in hard scatterings:



- Double hard parton scatts. O(3-100 GeV) should also take place. Seen?

Double Parton Scattering x-sections (p-p)

- Hard DPS provides direct info on transverse parton density-profile & correlations in proton:



- pQCD factorized expression for DPS x-section:

$$\sigma^{\rm DPS}_{(hh'\to ab)} = \left(\frac{m}{2}\right)\sum_{i,j,k,l}\int \Gamma^{ij}_{h}(x_1,x_2;\mathbf{b_1},\mathbf{b_2};Q_1^2,Q_2^2)\times\hat{\sigma}^{ik}_{a}(x_1,x_1',Q_1^2)\,\hat{\sigma}^{jl}_{b}(x_2,x_2',Q_2^2)$$
$$\times\Gamma^{kl}_{h'}(x_1',x_2';\mathbf{b_1}-\mathbf{b},\mathbf{b_2}-\mathbf{b};Q_1^2,Q_2^2)\,dx_1dx_2dx_1'dx_2'd^2b_1d^2b_2d^2b$$

Generalized PDFs = $f(x,Q^2,\mathbf{b})$

- Assumption 1: Factorization of transverse & longitudinal components

$$\Gamma^{ij}_{h}(x_1,x_2;\mathbf{b_1},\mathbf{b_2};Q_1^2,Q_2^2) = D^{ij}_{h}(x_1,x_2;Q_1^2,Q_2^2)\,f(\mathbf{b_1})f(\mathbf{b_2})$$

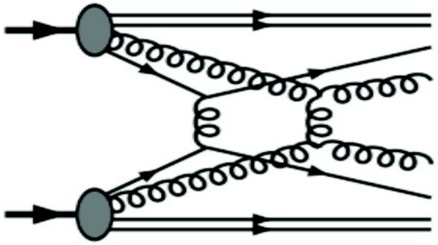
p-p overlap function: $$t(\mathbf{b}) = \int f(\mathbf{b_1})f(\mathbf{b_1}-\mathbf{b})d^2b_1$$

- Assumption 2: Double-PDF = product of 2 single PDF (no correlations)

$$D^{ij}_{h}(x_1,x_2;Q_1^2,Q_2^2) = D^{i}_{h}(x_1;Q_1^2)\,D^{j}_{h}(x_2;Q_2^2)$$

[Treleani, Diehl, Ryskin, Snigirev, Blok, Strikman, Gaunt, ...]

Double Parton Scattering x-sections (p-p)



- Hard DPS provides direct info on transverse parton density-profile & correlations in proton:
- pQCD factorized expression for DPS x-section:

$$\sigma^{DPS}_{(hh'\to ab)} = \left(\frac{m}{2}\right)\sum_{i,j,k,l}\int \Gamma^{ij}_{h}(x_1,x_2;\mathbf{b_1},\mathbf{b_2};Q_1^2,Q_2^2)\times\hat{\sigma}^{ik}_{a}(x_1,x_1',Q_1^2)\,\hat{\sigma}^{jl}_{b}(x_2,x_2',Q_2^2)$$
$$\times\Gamma^{kl}_{h'}(x_1',x_2';\mathbf{b_1}-\mathbf{b},\mathbf{b_2}-\mathbf{b};Q_1^2,Q_2^2)\,dx_1dx_2dx_1'dx_2'd^2b_1d^2b_2d^2b$$

Generalized PDFs = f(x,Q^2,**b**)

- Final simplified form for DPS x-section:

p-p overlap function

$$\sigma^{DPS}_{(hh'\to ab)} = \left(\frac{m}{2}\right)\frac{\sigma^{SPS}_{(hh'\to a)}\cdot\sigma^{SPS}_{(hh'\to b)}}{\sigma_{\rm eff}} \qquad \sigma_{\rm eff} = \left[\int d^2b\, t^2(\mathbf{b})\right]^{-1} \approx 13\pm 2\ \text{mb}$$

ISR,SppS Tevatron

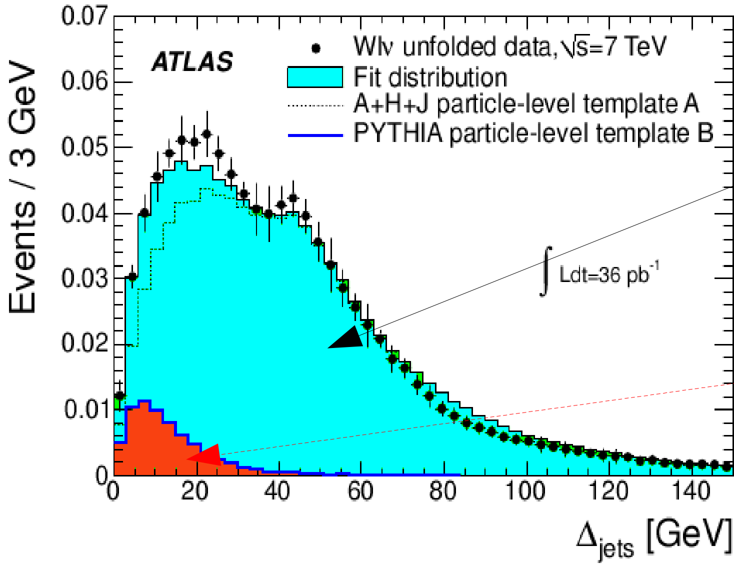
- Simplified parton transverse profiles in proton (CDF'97):

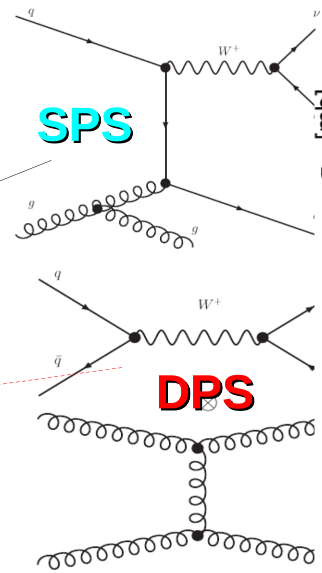
Effective DPS radius: r~0.3 – 0.7 fm smaller than e.m. one

Model for density	Form of density, dN/d^3r	Predictions rms r	Predictions $\sigma_{\rm eff}$	Measurements Scale (fm)
Solid sphere	Constant, $r<r_p$	$\sqrt{3/5}r_p$	~14.5 mb	$r_p=0.73$
Gaussian	$e^{-r^2/2\Sigma^2}$	$\sqrt{3}\Sigma$		$\Sigma=0.34$
Exponential	$e^{-r/\lambda}$	$\sqrt{12}\lambda$		$\lambda=0.20$
Fermi, $\lambda/r_0=0.2$	$(e^{(r-r_0)/\lambda}+1)^{-1}$	$1.07r_0$		$r_0=0.56$

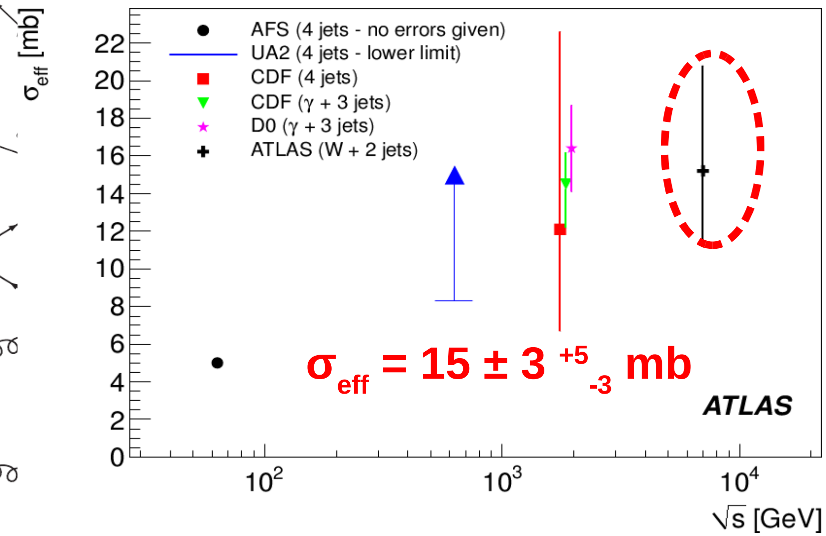
DPS searches at LHC: p-p → W^++2j

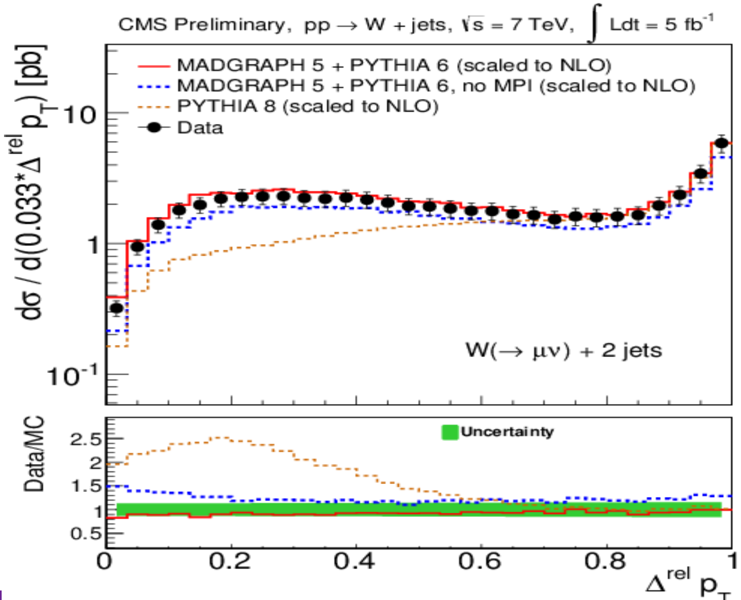
- Small signal in W+2jets events: Via di-jet p_T asymmetry (~0 in DPS)





ATLAS, arXiv:1301.6872





$$\Delta^{\rm rel}\, p_{\rm T} = \frac{|\vec{p}_T(j1) + \vec{p}_T(j2)|}{|\vec{p}_T(j1)| + |\vec{p}_T(j2)|}$$

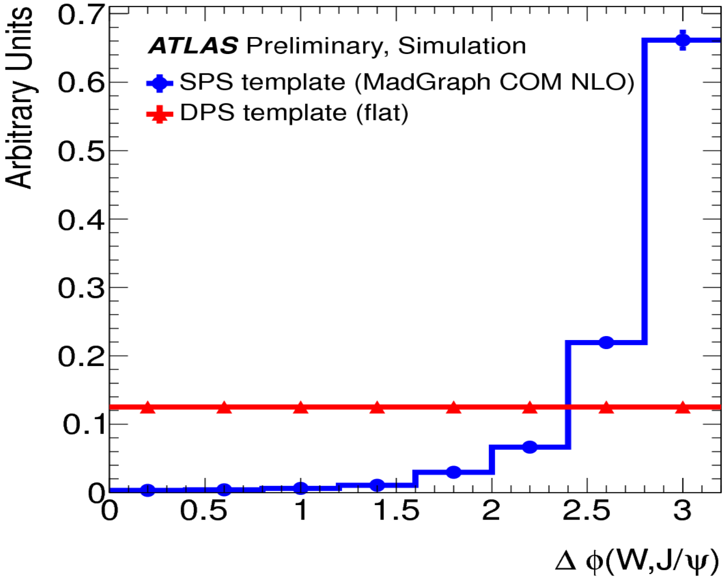
CMS, FSQ-12-028

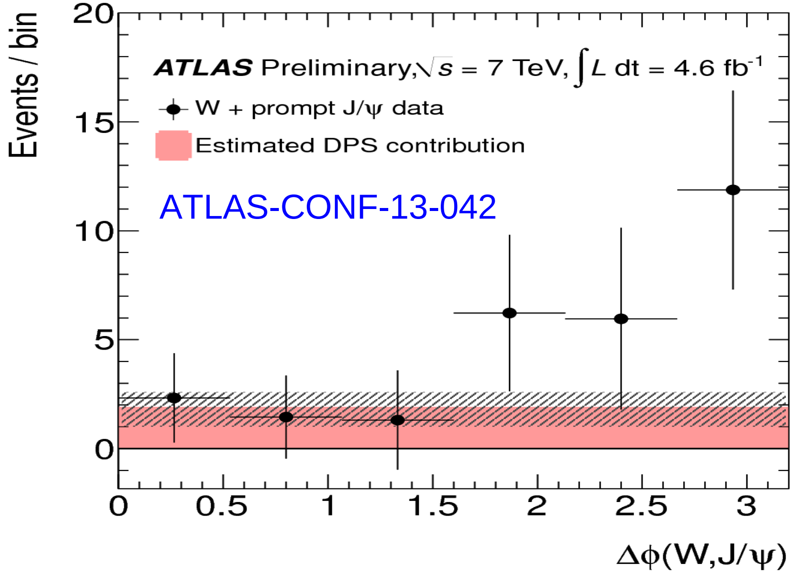
- Uncertainties on DPS extraction from:
  - Higher-order SPS contributions,
  - (technical) matching between ME hard jets & MPI jets

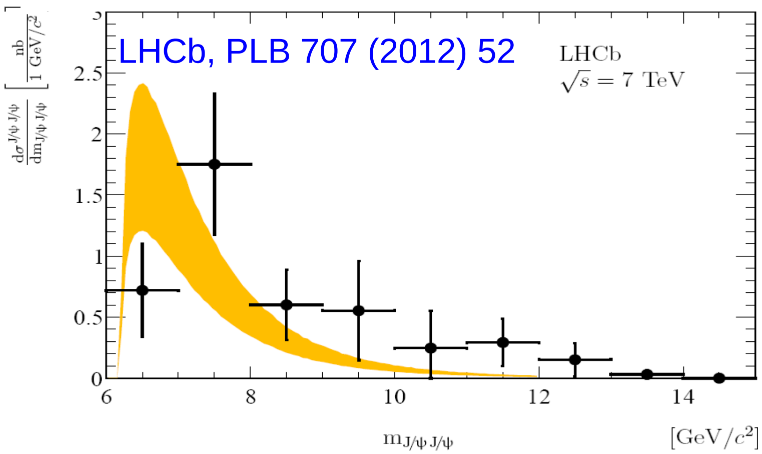
propagate into large uncertainty on σ_{eff}

DPS searches at LHC: p-p → W⁺+J/Ψ, J/Ψ J/Ψ

- Strong? signal in W+J/Ψ events: Via azimuthal W & J/Ψ correlation







- Less clear in double J/Ψ production:

  $\sigma_{J/\psi J/\psi}$ = 5.1±1.0± 1.1 nb

  $\sigma_{SPS}^{J/\psi\, J/\psi} + \sigma_{DPS}^{J/\psi\, J/\psi}$ ~ 4 nb + 2 nb = 6 nb

- Current TH uncertainties prevent from obtaining precise measurements of DPS fraction and σ_{eff} parameter at LHC.

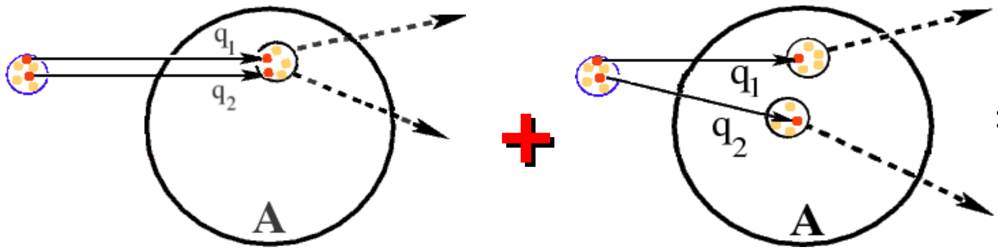
☛ Can p-Pb,Pb-Pb improve our understanding of DPS ?

Double Parton Scatterings in p-Pb at the LHC

Double Parton Scattering x-sections (p-Pb)

[DdE,Snigirev, arXiv:1211.0197]
[Treleani, Strikman, , ...]

- 2 contributions to DPS x-section in p-A:



$$\Rightarrow \sigma^{DPS}_{(pA)} = \sigma^{DPS,1}_{(pA)} + \sigma^{DPS,2}_{(pA)}$$

$$\sigma^{DPS,1}_{(pA\to ab)} = A \cdot \sigma^{DPS}_{(pN\to ab)} \quad + \quad \sigma^{DPS,2}_{(pA\to ab)} = \sigma^{DPS}_{(pN\to ab)} \cdot \sigma_{eff,pp} \cdot F_{pA}$$

p-A overlap function

$$F_{pA} = \int d^2r\, T^2_{pA}(\mathbf{r}) = 30.4 \text{ mb}^{-1}$$

Pb Woods-Saxon density (r=6.62 fm, a=0.546 fm)

► Factorized expression for DPS p-A x-section:

$$\sigma^{DPS}_{(pA\to ab)} = \left(\frac{m}{2}\right) \frac{\sigma^{SPS}_{(pN\to a)} \cdot \sigma^{SPS}_{(pN\to b)}}{\sigma_{eff,pA}}$$

$$\sigma_{eff,pA} = \frac{\sigma_{eff,pp}}{A + \sigma_{eff,pp} F_{pA}} = 21.5 \pm 1.1\ \mu b$$

(σ_{eff}=13±2mb)

► Ratio of DPS p-Pb/p-p x-sections: $\sigma_{eff,pp}/\sigma_{eff,pA} \approx 3A \approx 600$!

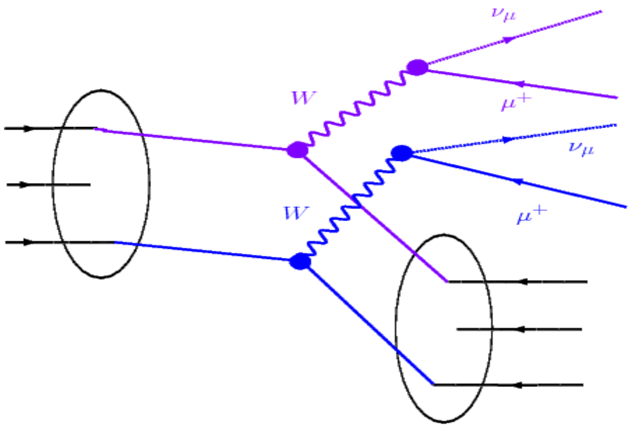
- DPS processes are large and can be unambiguously observed in p-A.
- Pb transverse density better known than proton. Determine $\sigma_{eff,pp}$?

DPS “golden channel”: p-p → W^+W^+,W^-W^-

[Kulesza, Stirling, Gaunt, Treleani, Del Fabbro, ...]

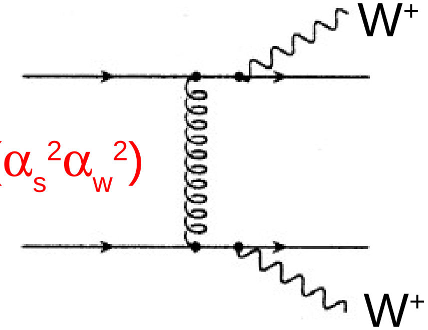
- Same-sign W-W production from 2 independent hard scatterings is an excellent DPS signature:

  - Well controlled pQCD x-sections.
  - Clean experimental final-state:
    2 like-sign leptons + missing-E_T

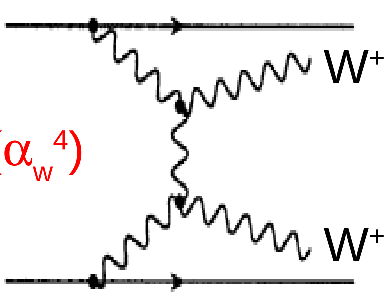


- Backgrounds: Same-sign W-W production in single parton scatterings (SPS) is higher-order and occurs only with 2 extra jets:

QCD ($\alpha_s^2\alpha_w^2$)



EWK (α_w^4)



- σ(WW,DPS)~1/3 σ(WWjj,SPS), but SPS background reducible by more than x20 applying jet cuts.

Case study: p-Pb → W^+W^+,W^-W^- at 8.8 TeV

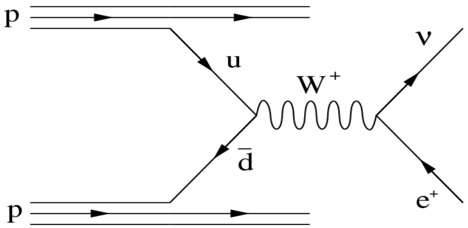
[DdE,Snigirev, arXiv:1211.0197]

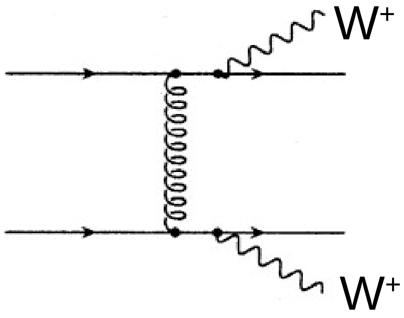
- Theoretical setup:

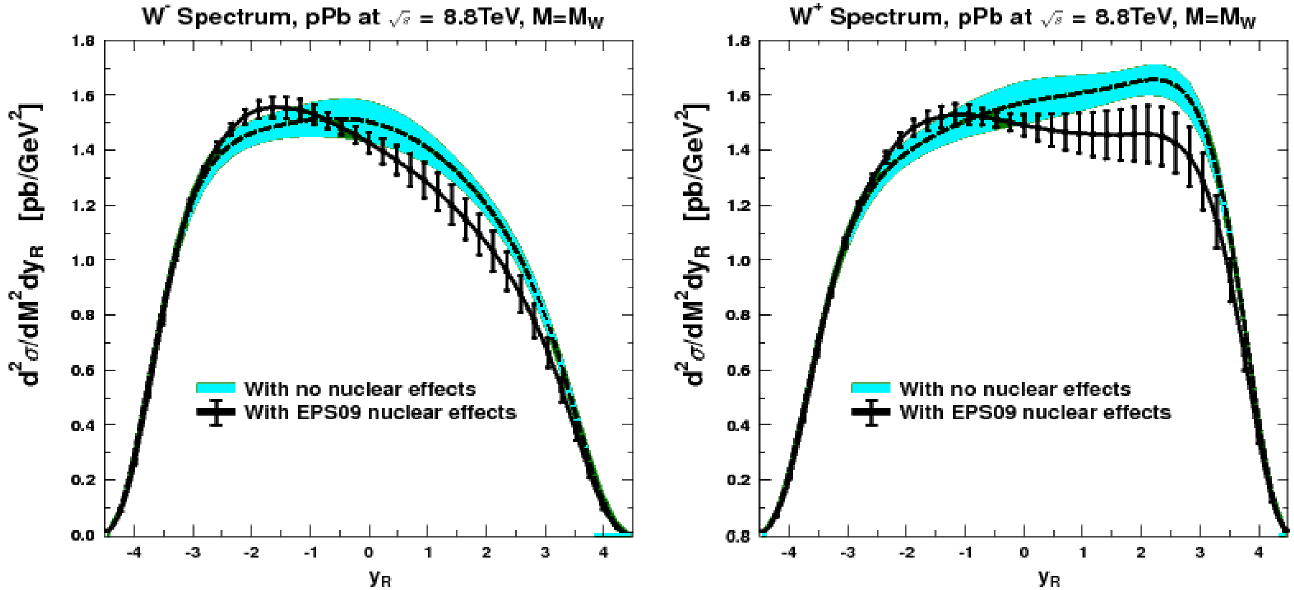
  ► MCFM 6.2: single-parton W^+,W^-
  W^+W^+jj (QCD) background

  - NLO accuracy.
  - Scales: $\mu(W) = m_W$, $\mu(WW)$ = 150 GeV
  - CT10 proton PDF, EPS09 Pb nuclear PDF:

  ~10% effects due nuclear (anti)shadowing alone:







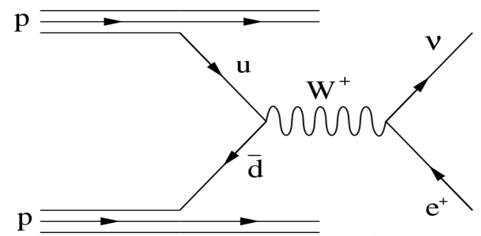
[Paukkunen&Salgado JHEP 1103 (2011) 071]

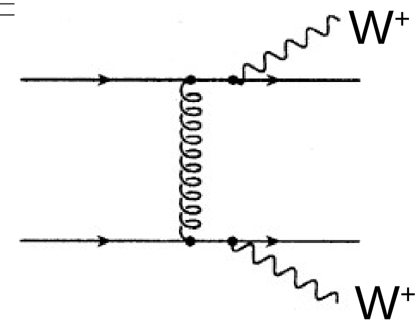
Isospin+shadow. effects on total inclusive x-sections:
W^- : +7%
W^+ : -15%
compared to p-p

Case study: p-Pb → W^+W^+,W^-W^- at 8.8 TeV

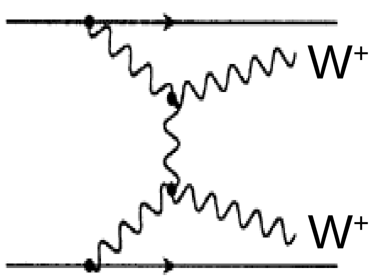
[DdE,Snigirev, arXiv:1211.0197]

- Theoretical setup:
  - MCFM 6.2: single-parton W^+,W^-
    W^+W^+jj (QCD) background
    - NLO accuracy.
    - Scales: $\mu(W) = m_W$, $\mu(WW)$ = 150 GeV
    - CT10 proton PDF, EPS09 Pb nuclear PDF
    - Uncertainties: ~10% (W)





  - VBFNLO 2.6.0: W^+W^+jj (EWK) background
    - NLO accuracy
    - Scales: $\mu^2 = t_{W,Z}$
    - CT10 PDF
    - Uncertainties: <10%



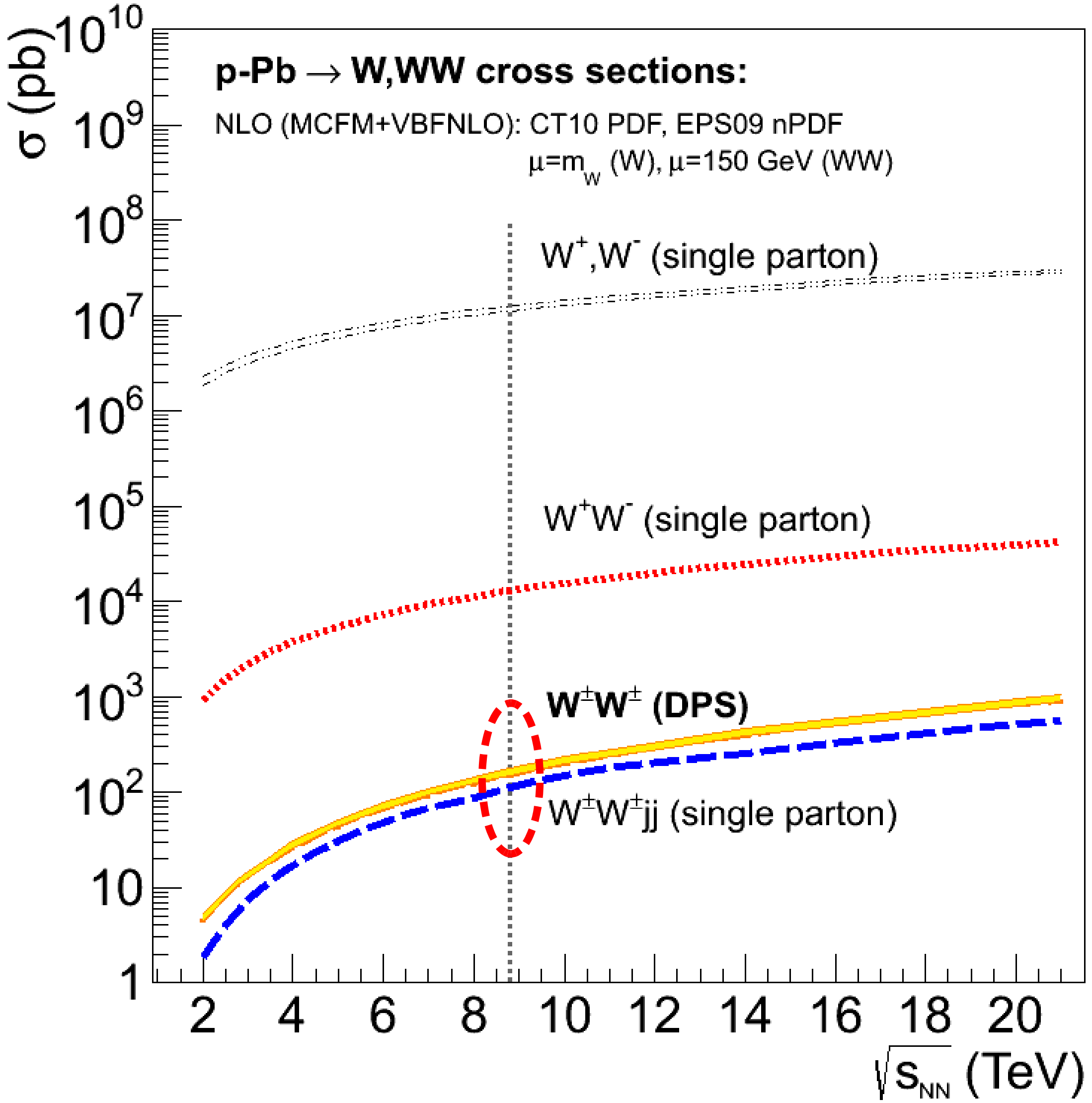
- x-sections in pb (signal & background):

p-Pb final-state: Code (process #): Order (σ units):	W^+ MCFM (1) NLO (μb)	W^- MCFM (6) NLO (μb)	W^+W^- MCFM (61) NLO (nb)	W^+W^+jj (QCD) MCFM (251) 'NLO' (pb)	W^+W^+jj (VBF) VBFNLO (250) NLO (pb)	$W^\pm W^\pm$ (DPS) Eq. (15) (pb)
$\sqrt{s_{NN}}$ = 5.0 TeV	6.85 ± 0.68	5.88 ± 0.59	5.48 ± 0.56	12.1 ± 1.2	12.4 ± 0.6	44. ± 8.
$\sqrt{s_{NN}}$ = 8.8 TeV	12.6 ± 1.3	11.1 ± 1.1	13.0 ± 1.3	40.4 ± 4.0	51.8 ± 2.0	152. ± 27.

Results: p-Pb → W^+W^+,W^-W^- at 8.8 TeV

[DdE,Snigirev, arXiv:1211.0197]

- Cross sections for all relevant SPS and DPS processes vs sqrt(s):



p-Pb @ 8.8 TeV:

σ(WW,DPS)$\approx$150 pb

σ(WWjj)$\approx$100 pb

±18% uncertainties:

±15% for σ_{eff}

±10% for scales&PDFs

Results: p-Pb → W^+W^+,W^-W^- at 8.8 TeV

[DdE,Snigirev, arXiv:1211.0197]

- Measurable final-states:
  - ► W's branching ratios:
    - BR(W→ lν) ~ 3 ×1/9, BR(W→ qq) ~ 2/3
    - Both leptonic: 4 final-states (μμ, ee, eμ, μe): $(4/9)^2$ ~ 1/20, 1/16 (+ τ)
    [1 leptonic + 1 hadronic (jet-charge): $(2/9 \times 4/3)^2 \sim 0.3$]
  - ► Typical ATLAS/CMS acceptances & efficiencies:
    - Leptons: |y|<2.5, p_T >15 GeV ⇒ ε_{WW} ~ 40%
- LHC p-Pb luminosities (note: very small pileup !):
  - ► $\mathcal{L}_{\rm int}$ = 0.2 – 2 pb^{-1} (increase to nominal p intensity, reduce beam size)
- Expected (purely leptonic) rates including yield loses & luminosity:

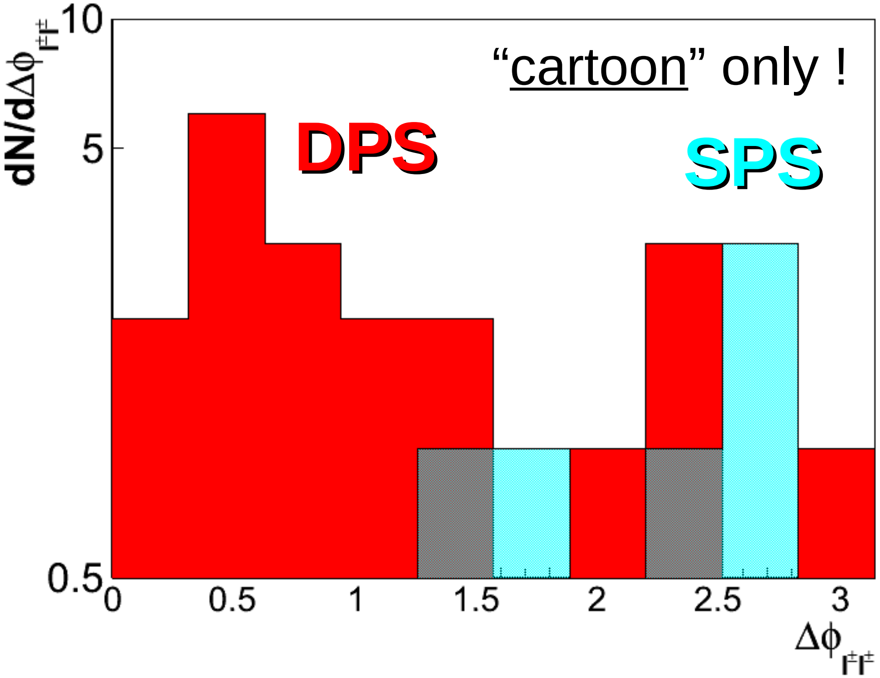
$$\mathcal{N} = \sigma^{\rm DPS}_{pPb\to WW}/(\varepsilon \cdot \mathcal{L}_{\rm int}) \approx$$ 1 – 10 same-sign WW pairs/year

(factor ~10 more in 1 lepton + 1-jet channel ?)

Results: p-Pb → W^+W^+,W^-W^- at 8.8 TeV

- Typical DPS-sensitive kinematical distributions for signal & background:

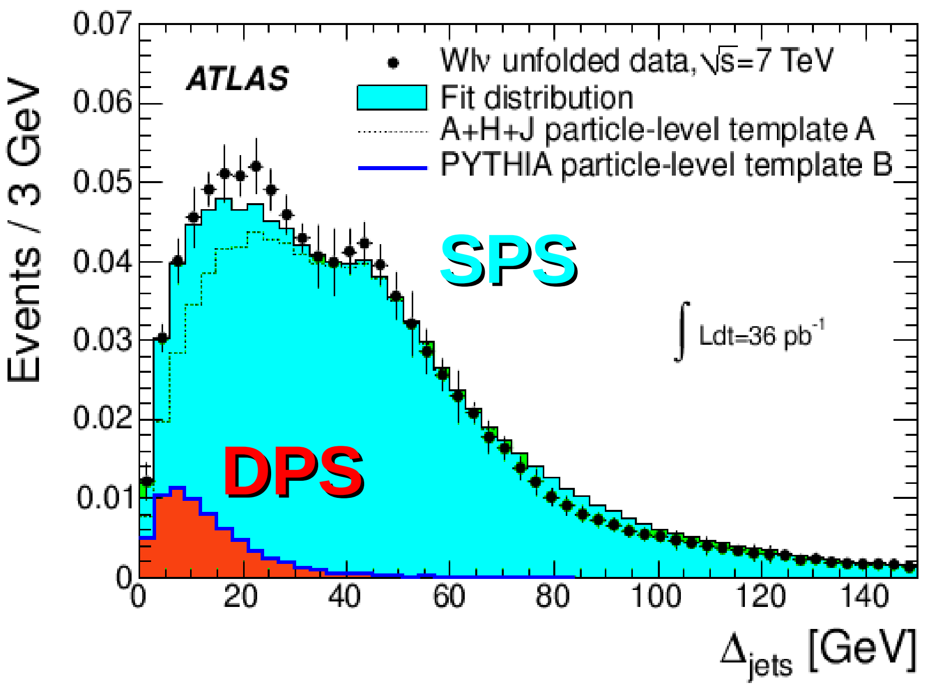
p-Pb @ 8.8 TeV (2 pb^{-1}):
Same-sign leptons
azimuthal separation:



N(DPS)≈10 evts   N(SPS)≈ 6 evts

Other reducible bckgds (WZ,$Z^{(*)}Z^{(*)}$,B^0B^0)
would need to be considered

Compare to:
p-p → W+2j @ 7 TeV (36 pb^{-1}):
dijet azimuthal separation

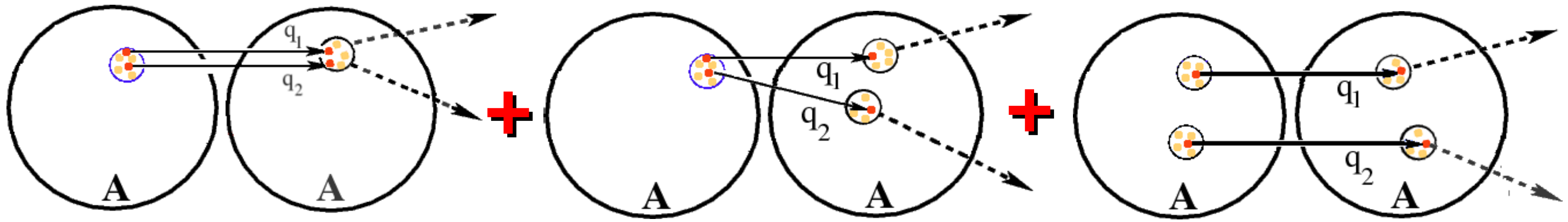


Double Parton Scatterings in Pb-Pb at the LHC

Double Parton Scattering x-sections (Pb-Pb)

[DdE,Snigirev, arXiv:1301.5845]

- 3 contributions to DPS x-section in A-A:



$$\sigma^{DPS,1}_{(AA\to ab)} = A^2 \cdot \sigma^{DPS}_{(NN\to ab)}$$

$$\sigma^{DPS,2}_{(AA\to ab)} \approx 2\sigma^{DPS}_{(NN\to ab)} \cdot \sigma_{eff,pp} \cdot A \cdot T_{AA}(0)$$

$$\sigma^{DPS,3}_{(AA\to ab)} \approx \sigma^{DPS}_{(NN\to ab)} \cdot \sigma_{eff,pp} \cdot A^2/\,2 \cdot T_{AA}(0)$$

  - ► Third "N_{coll} term", $\propto A^2 \cdot T_{AA}(0)$, clearly dominant (1:4:200 ratio)

- Factorized expression for DPS A-A x-section:

$$\sigma^{DPS}_{(AA\to ab)} = \left(\frac{m}{2}\right)\frac{\sigma^{SPS}_{(NN\to a)} \cdot \sigma^{SPS}_{(NN\to b)}}{\sigma_{eff,AA}}$$

$$\sigma_{eff,AA} = \frac{1}{A^2[\sigma^{-1}_{eff,pp} + \frac{2}{A}T_{AA}(0) + \frac{1}{2}T_{AA}(0)]} = 1.5\text{ nb}$$

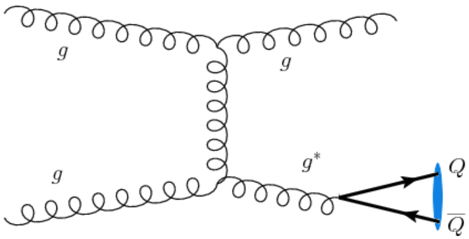
  - ► Ratio of DPS Pb-Pb/p-p x-sections: $\sigma_{eff,pp}/\sigma_{eff,AA} \propto A^{3.3}/5 \simeq 9\cdot 10^6$!

- Strong centrality dependence:

$$\sigma^{DPS}_{(AA\to ab)}[b_1,b_2] \approx \left(\frac{m}{2}\right)\sigma^{SPS}_{(NN\to a)} \cdot \sigma^{SPS}_{(NN\to b)} \cdot f_{\%}\sigma_{AA} \cdot \langle T_{AA}[b_1,b_2]\rangle^2$$

Case study: Pb-Pb → J/ψ J/ψ at 5.5 TeV

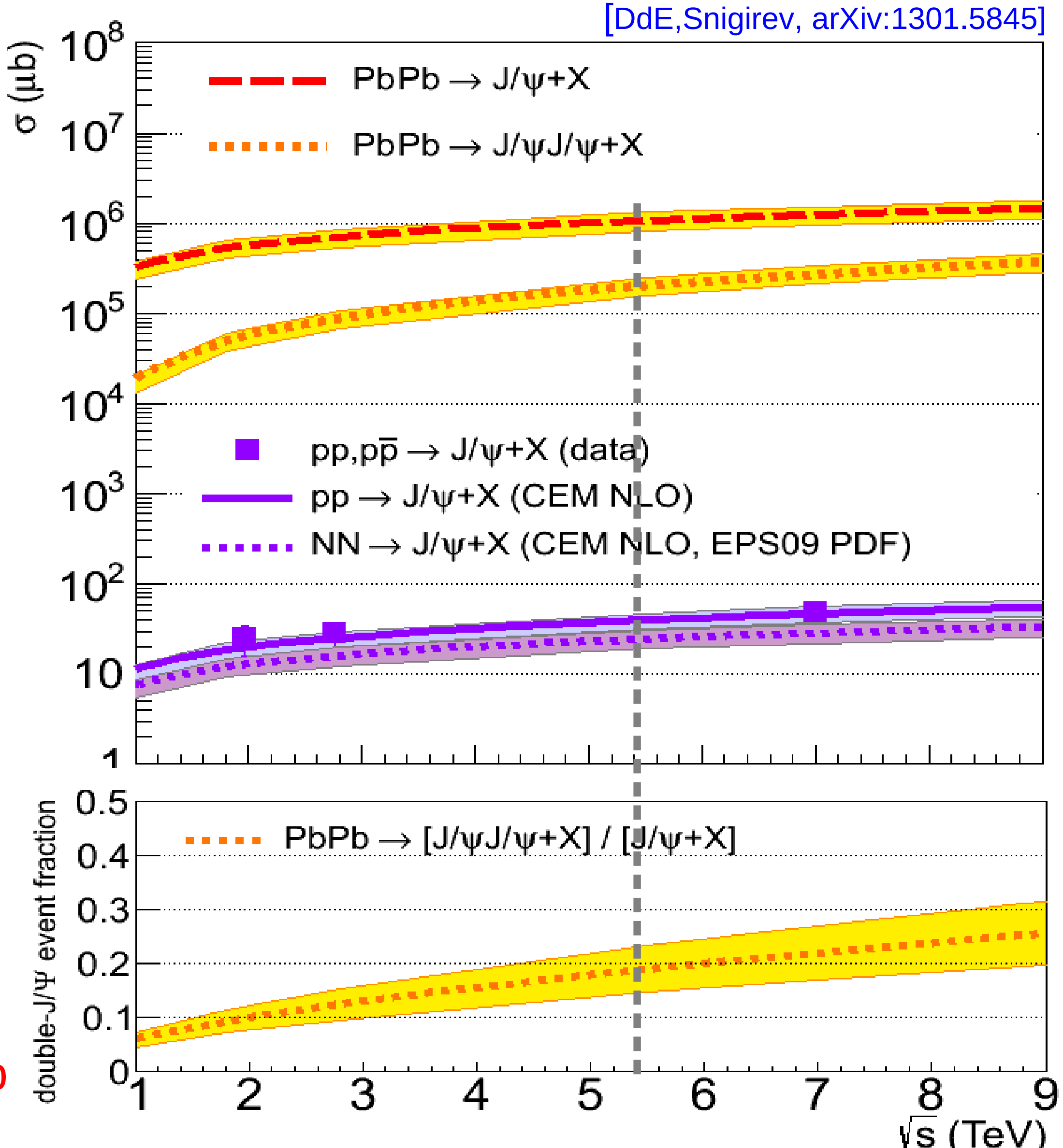
- FONLL+CEM (R.Vogt): Single-parton J/ψ



- NLO accuracy.
- Scales: $\mu_R = \mu_R = 1.5 \cdot m_c$
- Good agreement with Tevatron&LHC data
- EPS09 Pb nPDF

  20-35% shadowing x-section reduction

[DdE,Snigirev, arXiv:1301.5845]



- At 5.5 TeV:

$\sigma^{DPS}(\text{Pb-Pb} \rightarrow J/\psi\, J/\psi\, X)$ =200±50 mb

20% of Pb-Pb collisions have two J/ψ produced !

Results: Pb-Pb → J/ψ J/ψ at 5.5 TeV

[DdE,Snigirev, arXiv:1301.5845]

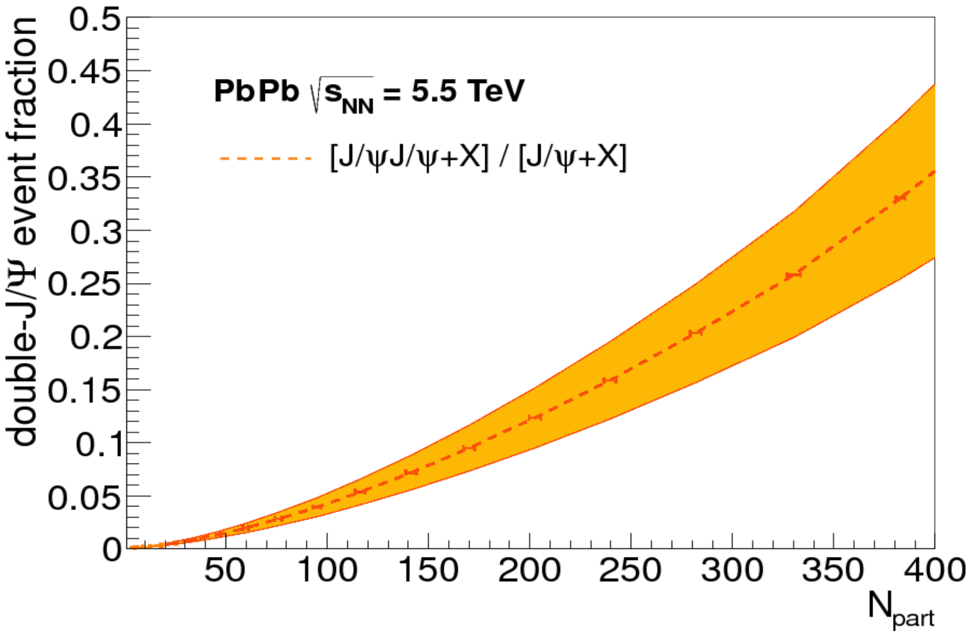
- Visible rates:
  - Reduced x-sections per unit-y: $d\sigma_{J/\psi}/dy \approx \sigma_{J/\psi}/8$
  - BR($J/\psi \to l^+l^-$) ≈ 6%
  - Typical ALICE/CMS acceptance & efficiencies: ε ≈ 1/12
- Expected dimuon rates including yield all loses & 1-nb^{-1} luminosity:

$$\mathcal{N} = \sigma^{\rm DPS}_{\text{Pb-Pb} \to J/\psi\, J/\psi'} / (\varepsilon \cdot \mathcal{L}_{\rm int}) \approx$$ 240 double-J/ψ per year

(factor 2 less including final-state suppression)

- Centrality dependence of double-J/ fraction: 35% of central Pb-Pb collisions have two J/ψ produced !

Seeing 2 J/ψ on event-by-event basis not to be blindly taken as signal of c-cbar recombination.



Summary

- MPI are crucial to understand p-p collisions at LHC: MB, UE ...
- MPI $\Rightarrow$ Double <u>hard</u> parton scatterings (pQCD framework available)

$$\sigma^{\rm DPS}_{(hh'\to ab)} = \left(\frac{m}{2}\right) \frac{\sigma^{\rm SPS}_{(hh'\to a)} \cdot \sigma^{\rm SPS}_{(hh'\to b)}}{\sigma_{\rm eff}}$$

All details on proton transverse parton density/correlations encoded into $\sigma_{\rm eff}$ parameter

- We derived DPS x-sections "pocket formula" for p-A and A-A:

$$\sigma_{\rm eff,pA} = \frac{\sigma_{\rm eff,pp}}{A + \sigma_{\rm eff,pp}\,F_{\rm pA}} = 21.5 \pm 1.1\ \mu{\rm b}$$

$$\sigma_{\rm eff,AA} = \frac{1}{A^2[\sigma^{-1}_{\rm eff,pp} + \frac{2}{A}T_{AA}(0) + \frac{1}{2}T_{AA}(0)]} = 1.5\ {\rm nb}$$

Huge enhancements ! $\sigma_{\rm eff,pp}/\sigma_{\rm eff,pA} \approx 3\,A \approx 600$ $\quad \sigma_{\rm eff,pp}/\sigma_{\rm eff,AA} \propto A^{3.3}/5 \simeq 9 \cdot 10^6$

- DPS in p-A can help determine $\sigma_{\rm eff,pp}$
  p-Pb $\rightarrow$ W^+W^+, W^-W^-, NLO+nuclear PDFs for signal & bckgds.
  σ(same-sign WW,DPS) ≈ 150 pb (10 counts/year).

- DPS in A-A are clearly dominated by binary-scaling term.
  Pb-Pb $\rightarrow$ J/ψJ/ψ, NLO+nuclear PDFs.
  dσ(J/ψJ/ψ,DPS)/dy ≈ 200 mb (240 counts/year in dimuon channel)

DPS p-Pb x-sections: Other examples

- 2013 run:
  $\sqrt{s}$ = 5.02 TeV, L_{int} = 34 nb^{-1}

- Nominal run:
  $\sqrt{s}$ = 8.8 TeV, L_{int} = 0.2 – 2 pb^{-1}

DPS cross sections (NLO):

2013 run	Nominal run
$\sigma_{DPS}(J/\psi + J/\psi) \approx 11\ \mu b$	$\sigma_{DPS}(J/\psi + J/\psi) \approx 22\ \mu b$
$\sigma_{DPS}(J/\psi + W) \approx 63$ nb	$\sigma_{DPS}(J/\psi + W) \approx 164$ nb
$\sigma_{DPS}(J/\psi + Z) \approx 3$ nb	$\sigma_{DPS}(J/\psi + Z) \approx 8$ nb
$\sigma_{DPS}(ssWW) \approx 44$ pb	$\sigma_{DPS}(ssWW) \approx 152$ pb

Visible DPS yields (after BRs×acc.×effic):

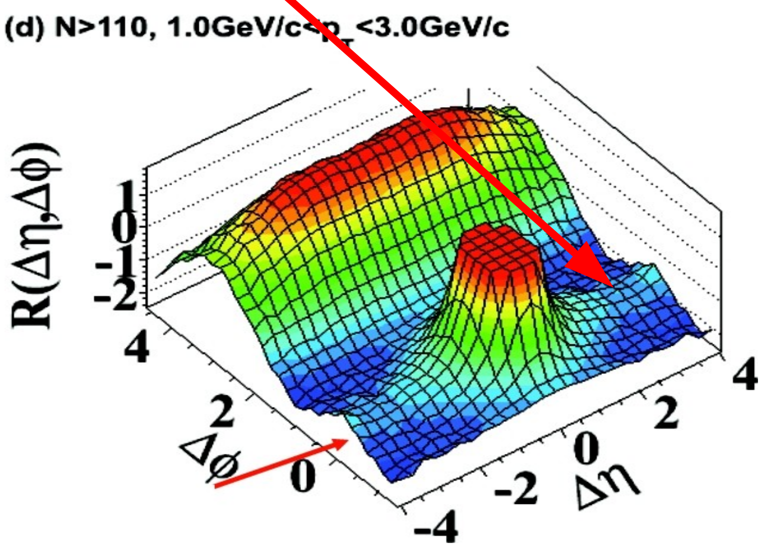
2013 run	Nominal run
$N_{DPS}(J/\psi + J/\psi) \approx 2$	$N_{DPS}(J/\psi + J/\psi) \approx$ 24 – 240
$N_{DPS}(J/\psi + W) \approx 2.6$	$N_{DPS}(J/\psi + W) \approx$ 40 – 400
$N_{DPS}(ssWW) \approx 0.2$	$N_{DPS}(ssWW) \approx$ 1 – 10
$N_{DPS}(J/\psi + Z) \approx 0.1$	$N_{DPS}(J/\psi + Z) \approx$ 0.5 – 5

- Many double hard scatterings processes visible in p-Pb !
  Valuable constraints on parton transverse size & correlations

Backup slides

MPI evidence (LHC, p-p "central"): "ridge" ?

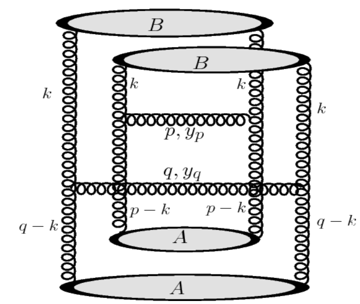
- Observation of long-range (over $\Delta\eta$~8 !) near-side hadron correlations: "Ridge" in "central" (high multiplicity) p-p collisions



[CMS, JHEP 1009 (2010) 091]

- Interpretations:
  - Correlated gluons in initial-state ?

$$|\mathbf{k}_\perp| \sim |\mathbf{p}_\perp - \mathbf{k}_\perp| \sim |\mathbf{q}_\perp \pm \mathbf{k}_\perp| \sim Q_s$$



Multi-parton-interactions: α_s^8 enhancement of near-side diagram

[R.Venugopalan et al.]

  - Final-state collective parton-flow ?
    PYTHIA + β_T=0.5 generates structure

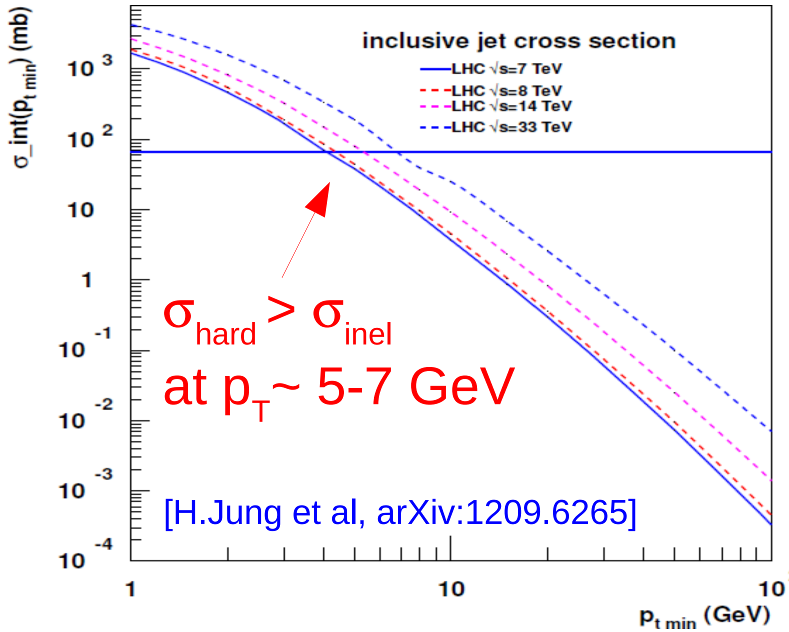
- Remains an intriguingly large effect without clearcut explanation currently
- Enhanced ridge also observed in p-Pb at 5 TeV

Multi-parton interactions in p-p

- pQCD (mini)jet production x-section is bigger than total inel p-p x-section for p_{Tmin}~ 5-7 GeV at the LHC !

$$\sigma_{\mathrm{hard}}(p_{\perp\mathrm{min}}) = \int_{p^2_{\perp\mathrm{min}}}^{s/4} \frac{\mathrm{d}\sigma}{\mathrm{d}p^2_\perp}\,\mathrm{d}p^2_\perp$$

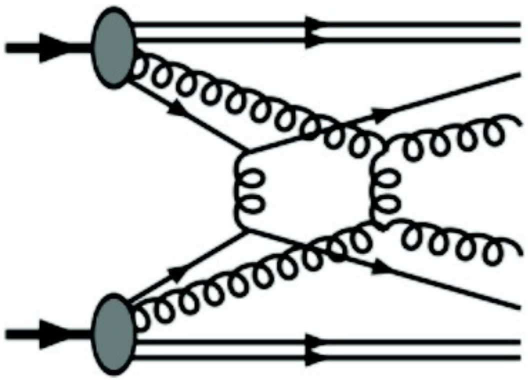
... Why this happens ?



- Very high gluon densities at small-x.
- Solution (2): Multi-parton interactions

Interpret $$\langle n \rangle = \frac{\sigma_{\mathrm{hard}}(p_{\perp\mathrm{min}})}{\sigma_{\mathrm{inel}}}$$

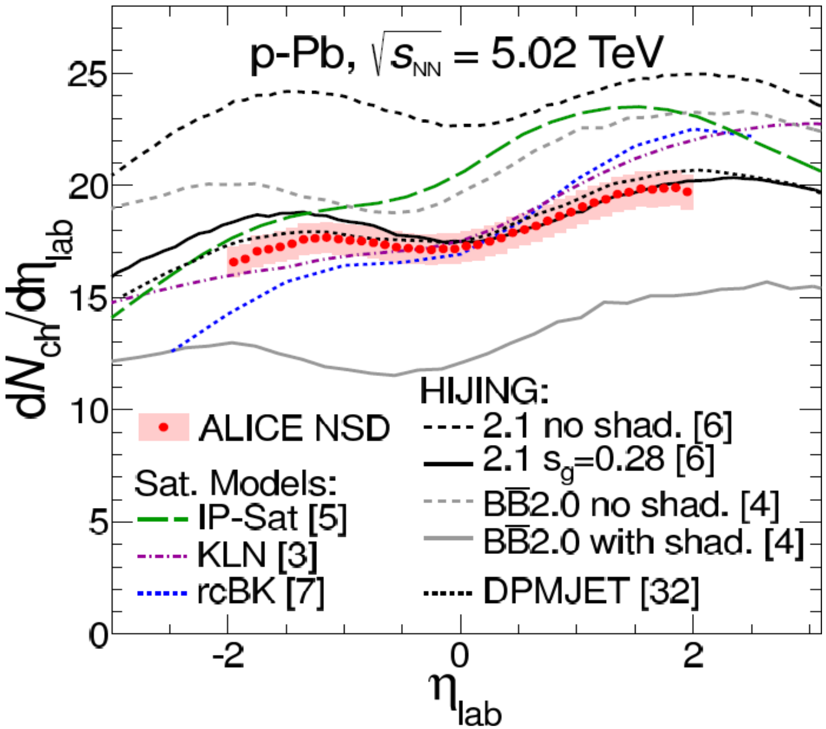
= average number of parton–parton scatterings above $p_{\perp\mathrm{min}}$ in an event



- PYTHIA, HERWIG include them via transverse parton density profile.

MPI evidence (LHC, p-Pb): inclusive hadron prod.

[ALICE, arXiv:1210.3615]



- Inclusive hadron production in p-Pb at 5.02 TeV:

  Models need shadowing/ saturation of Pb gluon PDF to reproduce the data

- Center-of-mass dependence:

  Power-law exponent: n~0.10

  Naive expectations:
  Gluon sat: n~0.11-0.14
  Pure RFT: n~0.10
  Minijets: n~0.15